Patterns in Stock Market Returns?

A new methodology for investigating anomalous returns after simple price patterns

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Abstract

The Weak Form of the Efficient Market Hypothesis says that it should not be possible to predict future price movements based on information contained in past prices. Therefore, it should also not be possible to construct a profitable trading system based on price patterns or information contained in past price movements. We developed a methodology to extract a baseline return from a sample of US equities and then investigated price movement following patterns from the traditional Technical Analysis literature, and large single-day price changes in stocks. We found significant anomalies associated with some of the patterns which, if they are stable through time and in other samples of stocks, may offer potentially profitable trading opportunities.

We then constructed AR, EGARCH, and GARCH-M price models and examined these same patterns on the EMH models. We found significant differences between the models and real prices, and found that the anomalies in price movements were not completely explained by the EMH models.

Price Patterns

Patterns in Prices?

Technical Analysis is often described as the art and science of predicting future price movements based on patterns in past prices. Adherents claim that all known information about a security is somehow encoded within the price, thus they claim be able to forecast future price direction and magnitude with varying degrees of reliability based on past prices.

The field is sometimes very light on "science" – patterns are vague and untestable, eccentric connections have been suggested (sunspot cycles, astrology, patterns based on Biblical scriptures, quasi-mystical ratios and proportions) and writers have made many unsubstantiated claims. In addition, many of the non-academic studies of Technical Analysis suffer from serious problems – sample sizes are too small (a recent book published in 2002 analyzes trading "systems" which have a total of 7 to 22 trades) and many writers seem to have a weak grasp of basic inferential statistics.

The academic community, on the other hand, holds to the Efficient Market Hypothesis, some forms of which assert that it is not possible to consistently make above-average profits in the market through any methodology. Technical Analysis, with its unverified track record and vague terminology, is easily dismissed as just another form of black magic. Our intent in doing this research was to try to bridge that gap—we wanted to rigorously define price patterns in the market, subject them to rigid statistical tests, and to evaluate the results impartially.

A New Methodology for Investigating Price Patterns

Technical Analysis tends to define patterns in vague and confusing ways. It is difficult to evaluate a pattern such as a "descending V bottom" or a "head and shoulders top" (though pattern recognition like this is possible with neural networks), so we chose patterns for this research that are simple and concrete. We first wanted to evaluate some patterns that are often claimed to be profitable patterns in the vernacular Technical Analysis literature. We also coded and tracked the simple technical trading rules investigated by

Brock, Lakonishok and LeBaron in 1992[1], tracked the performance of those systems to the current date (though we do not present the results in this paper), and investigated the technical patterns they identified as profitable trading patterns. We were also guided by our own trading experience and wanted to evaluate some of the patterns that we have used profitably in our own trading. The patterns we have chosen to evaluate in this study are: [redacted], [redacted], [redacted] moving average crossovers, channel breakouts and moving average slope changes.

Our methodology was as follows: We first chose a random sample of 300 stocks from the current Russell 3000. The Russell 3000 was chosen because it is a broad market index representing a mix of large and small cap stocks (though very small "micro-cap" stocks are not represented in this sample). The sample was checked for data integrity (in particular, we eliminated stocks with very small daily trading volumes or short histories), and we trimmed the sample to the 10 years from 11/1/1996 to 11/1/2006.

We then ran a random entry system on each stock (entry criteria was simply any day the system is flat, there is a 5% chance of buying the market on the close of the current day) and tracked the percent change from the entry point each day for 100 days. After 100 days, the system went flat and allowed a new entry according to the 5% random entry rule. This system was looped through the entire sample of 300 stocks several times until we had accumulated 25,000 trades. The intent was to find the baseline drift return for each sample for that particular time window (evaluating different stocks and/or changing the time period under consideration would require recalibrating this baseline return).

Figure 1. Baseline Mean Return and Standard Deviation

days	μ_B	σ_B
0	0.1%	2.55%
1	0.15%	4.24%
2	0.22%	5.12%
3	0.34%	5.87%
4	0.41%	6.48%
5	0.45%	7.08%
10	0.80%	9.23%
15	1.11%	11.06%
20	1.51%	12.40%
25	1.86%	13.71%
30	2.09%	15.06%
35	2.39%	16.11%
40	2.68%	17.23%
45	2.88%	17.75%
50	3.11%	18.54%
55	3.33%	19.27%
60	3.68%	19.85%
65	4.02%	20.57%
70	4.25%	21.24%
75	4.48%	21.70%
80	4.64%	22.25%
85	4.91%	22.52%
90	5.24%	22.98%
95	5.48%	23.30%
99	5.76%	23.65%

We trimmed outliers at the 99.5 percentile and show the baseline results in Figure 1. One might immediately question why the returns are so high. We believe it is because we captured a period of exceptional volatility in small cap stocks (the dot.com bubble and subsequent bear market), followed by a period of unusually strong small cap returns. In addition, the sample was drawn from the *current* Russell 3000 and so by definition suffers from survivorship bias, and our attempts to only include actively traded stocks probably intensified this effect. More research is needed to explore price action in very small-cap and low-volume stocks, but the high returns should not affect the validity of this study. This baseline is a "hurdle rate"—any technical pattern needs to show a statistically significant excess return compared to this baseline to be considered valid.

We also considered other ways of calculating the baseline mean return. We wanted to directly answer the question "what return would an investor have seen from randomly buying stocks in this sample?" We could easily have calculated an average daily return for the sample (and, in fact, did that and found it was virtually the same as our bootstrapped

[1] Brock, Lakonishok, LeBaron, *Simple Technical Trading Rules and the Stochastic Properties of Stock Returns,* Journal of Finance, December 1992.

returns), but since the technology was available to allow us to do the random trades, we wanted to simulate the trading environment as completely as possible. We ran other sets of random entries on the sample (different time windows, different probabilities of entry) and found in all cases the means did not differ significantly from our baseline.

The decision to trim outliers was a difficult one. Conversations with traders suggest that many traders make most of their profits from 2-3 "good" trades out of a hundred "average" trades. This would seem to indicate that traders may make their profits from outliers, and that at least some technical trading strategies may depend on outliers for their profits. We looked at the data both with and without outliers (and also trimming outliers at the 99th and 95th percentile) and decided that trimming at the 99.5th percentile was a reasonable compromise. Most of the patterns we looked at showed 1 or 2 trades out of 10,000 that had 5 to 10 times the return of the next smallest trade. It seemed appropriate to remove these most extreme outliers while preserving some of the less extreme weight in the tails. (When we examined the data with outliers it was also necessary to recalculate the baseline with outliers included.) Though we do not present detailed information here, many of the patterns we dismiss as lacking significance may achieve statistical significance when outliers are included.

"Common Practice" Technical Patterns

Once the baseline was calculated, we were able to code simple trading systems that entered the market on a specific price pattern, record returns for each of 100 days, and compare the pattern returns to the baseline. There are two issues to consider: statistical significance was determined using Welch's t-test to compare the pattern mean to the baseline mean, but practical significance was more difficult to assess. For instance, one pattern we examined showed a t-value of 3.33 on a specific day, certainly meeting the criteria for statistical significance, but the mean difference from the baseline was only .65%. Execution ability certainly plays a part in profitable trading, and our experience leads us to believe that executions in liquid stocks will probably cost between .2% - .5% on *each side* of the trade. This return is probably not economically significant.

We first examined common technical patterns used in the literature. Moving average crossovers are used in many trading systems. A buy signal is given when a fast (shorter period) moving average crosses above the value of a slower moving average. Many different values are in common use, but financial journalists commonly refer to the 50 and 200 day moving averages, so we began with these parameters. The pseudo-code for the moving average system is:

```
Moving average crossover buy:
FastMA = average of past 50 days' closing prices
SlowMA = average of past 200 closes

Buy close of this bar if
      Market position = flat and
      FastMA > SlowMA and
      FastMA[previous bar] <= SlowMA[previous bar]
```

Figure 2. Results of 50/200 Day Moving Average Crossovers

	50/200 Day Moving Average Buy				50/200 Day Moving Average Sell			
	$\mu_X - \mu_B$	df	t-val	p-val	$\mu_X - \mu_B$	df	t-val	p-val
1	(0.18%)	1,969	2.09	0.0371	(0.08%)	1,792	0.74	0.4612
2	(0.18%)	1,802	1.53	0.1260	(0.09%)	1,723	0.63	0.5313
3	(0.22%)	1,767	1.59	0.1118	0.08%	1,710	(0.47)	0.6398
4	(0.38%)	1,773	2.49	0.0128	0.16%	1,654	(0.82)	0.4105
5	(0.31%)	1,746	1.82	0.0692	0.18%	1,662	(0.83)	0.4061
10	(0.28%)	1,685	1.16	0.2482	0.06%	1,686	(0.24)	0.8080
15	(0.22%)	1,690	0.77	0.4423	(0.07%)	1,705	0.23	0.8204
20	(0.29%)	1,697	0.90	0.3675	(0.47%)	1,675	1.30	0.1952
25	(0.51%)	1,678	1.38	0.1672	(0.72%)	1,685	1.85	0.0651
30	(0.95%)	1,715	2.47	0.0134	(0.64%)	1,694	1.51	0.1321
35	(0.95%)	1,688	2.23	0.0257	(0.97%)	1,666	2.03	0.0422
40	(1.23%)	1,691	2.73	0.0065	(1.01%)	1,701	2.11	0.0350
45	(1.44%)	1,675	3.02	0.0026	(1.09%)	1,694	2.18	0.0296
50	(1.49%)	1,666	2.96	0.0032	(0.94%)	1,686	1.78	0.0748
55	(1.61%)	1,671	3.09	0.0020	(0.83%)	1,681	1.50	0.1336
60	(1.58%)	1,666	2.92	0.0036	(0.99%)	1,683	1.74	0.0815
65	(1.73%)	1,672	3.11	0.0019	(1.10%)	1,695	1.90	0.0576
70	(1.57%)	1,671	2.74	0.0062	(1.30%)	1,701	2.19	0.0286
75	(1.35%)	1,660	2.26	0.0240	(1.05%)	1,699	1.73	0.0830
80	(1.11%)	1,664	1.82	0.0686	(0.85%)	1,694	1.35	0.1767
85	(1.14%)	1,649	1.80	0.0713	(1.10%)	1,694	1.73	0.0839
90	(1.10%)	1,642	1.69	0.0921	(1.47%)	1,709	2.33	0.0198
95	(0.86%)	1,626	1.27	0.2056	(1.48%)	1,709	2.31	0.0211
99	(0.76%)	1,620	1.09	0.2774	(1.76%)	1,714	2.73	0.0063

Figure 2 shows the results of the test. We can conclude that there is no significant edge to the 50/200 moving average crossover in the sample we examined. P-values for the buys were quite high, but it is also important to note that the mean difference from the baseline was *negative*. This is exactly the opposite of what "should" happen with the system, since a profitable system should generate returns higher than the baseline for buys. For the sell side, the mean returns were also negative, but most of the p-values are too high to be considered significant. We repeated the work with a number of different moving average periods. The results did not differ significantly from what we have already presented – we find no repeatable edge to buying or selling moving average crosses.

We next looked at channel breakouts. This is another technique that is common in the literature and daily news ("XYZ just broke above its yearly highs" etc.) and anecdotal evidence suggests that many CTAs were using channel breakout systems through the 1980's and mid 1990's. There are many variants of this system, but we chose to require closing confirmation (the current day must close above the previous N day high, at which point we buy the close of the current day). We also included a filter that prevented buying continued runs in trends.

```
N-Day Channel Breakout System
      Buy close of current day if
            Close > Highest(H of past N days)[1]
            Close [1] <= Highest (H of past N days)[2]
      Sell close of current day if
            Close < Lowest(L of past N days)[1]
            Close[1] >= Lowest(L of past N days)[2]
```

Figure 3. Channel breakout Buy Returns

	10 day breakout up				20 day breakout up				50 day breakout up				Yearly Breakout Up			
days	$\mu_X - \mu_B$	df	t-val	p-val	$\mu_X - \mu_B$	df	t-val	p-val	$\mu_X - \mu_B$	df	t-val	p-val	$\mu_X - \mu_B$	df	t-val	p-val
1	(0.02%)	8,762	0.37	0.7094	(0.04%)	7,572	0.59	0.5579	0.00%	3,914	(0.02)	0.9809	0.20%	2,251	(2.32)	0.0203
2	(0.04%)	8,197	0.43	0.6668	(0.02%)	6,913	0.21	0.8320	0.01%	3,570	(0.13)	0.8969	0.07%	2,182	(0.63)	0.5314
3	(0.02%)	8,235	0.19	0.8515	(0.06%)	6,580	0.54	0.5876	(0.11%)	3,710	1.00	0.3184	0.04%	2,175	(0.33)	0.7417
4	(0.07%)	8,135	0.64	0.5249	(0.18%)	6,685	1.64	0.1020	(0.14%)	3,716	1.18	0.2369	(0.04%)	2,165	0.26	0.7928
5	(0.07%)	8,074	0.58	0.5653	(0.19%)	6,800	1.60	0.1101	(0.16%)	3,751	1.19	0.2335	(0.04%)	2,156	0.27	0.7886
10	(0.37%)	8,152	2.53	0.0115	(0.28%)	6,693	1.73	0.0828	(0.27%)	3,656	1.53	0.1258	(0.12%)	2,063	0.57	0.5686
15	(0.39%)	8,049	2.15	0.0316	(0.17%)	6,745	0.91	0.3638	(0.04%)	3,617	0.17	0.8679	0.25%	2,010	(0.93)	0.3520
20	(0.38%)	7,940	1.89	0.0586	(0.16%)	6,602	0.74	0.4600	(0.15%)	3,594	0.60	0.5457	(0.20%)	1,994	0.65	0.5180
25	(0.37%)	7,822	1.61	0.1068	(0.32%)	6,601	1.34	0.1805	(0.15%)	3,626	0.55	0.5854	(0.23%)	2,015	0.69	0.4878
30	(0.29%)	7,875	1.17	0.2422	(0.39%)	6,663	1.48	0.1383	0.09%	3,521	(0.29)	0.7710	(0.00%)	2,012	0.01	0.9894
35	(0.45%)	7,864	1.69	0.0904	(0.47%)	6,581	1.67	0.0959	0.06%	3,503	(0.19)	0.8467	0.21%	2,009	(0.54)	0.5866
40	(0.34%)	7,822	1.20	0.2302	(0.31%)	6,591	1.01	0.3111	0.24%	3,479	(0.68)	0.4992	0.54%	2,009	(1.29)	0.1964
45	(0.41%)	7,723	1.38	0.1668	(0.35%)	6,514	1.10	0.2694	0.18%	3,370	(0.47)	0.6398	0.51%	1,936	(1.08)	0.2795
50	(0.28%)	7,572	0.88	0.3763	(0.02%)	6,484	0.05	0.9595	0.19%	3,423	(0.48)	0.6300	0.79%	1,956	(1.64)	0.1004
55	(0.12%)	7,519	0.36	0.7223	(0.08%)	6,472	0.23	0.8170	0.05%	3,454	(0.13)	0.8999	0.79%	1,954	(1.58)	0.1149
60	(0.06%)	7,516	0.18	0.8556	(0.14%)	6,486	0.39	0.6942	(0.27%)	3,462	0.66	0.5097	0.72%	1,948	(1.38)	0.1675
65	(0.31%)	7,602	0.89	0.3726	(0.35%)	6,486	0.97	0.3328	(0.35%)	3,444	0.81	0.4157	0.86%	1,894	(1.48)	0.1393
70	(0.33%)	7,601	0.92	0.3570	(0.44%)	6,536	1.16	0.2443	(0.42%)	3,455	0.95	0.3429	1.09%	1,911	(1.86)	0.0624
75	(0.27%)	7,663	0.74	0.4610	(0.50%)	6,525	1.31	0.1917	(0.41%)	3,484	0.91	0.3603	0.97%	1,930	(1.67)	0.0950
80	(0.20%)	7,723	0.53	0.5977	(0.38%)	6,484	0.96	0.3375	(0.44%)	3,473	0.96	0.3362	1.19%	1,924	(1.98)	0.0480
85	(0.15%)	7,707	0.39	0.6969	(0.25%)	6,402	0.62	0.5364	(0.30%)	3,425	0.64	0.5201	1.01%	1,929	(1.67)	0.0952
90	(0.13%)	7,685	0.33	0.7406	(0.37%)	6,450	0.89	0.3740	(0.40%)	3,419	0.82	0.4133	0.73%	1,952	(1.23)	0.2186
95	(0.27%)	7,665	0.69	0.4901	(0.37%)	6,425	0.88	0.3806	(0.59%)	3,407	1.20	0.2299	0.82%	1,959	(1.37)	0.1707
99	(0.33%)	7,709	0.82	0.4120	(0.38%)	6,390	0.88	0.3798	(0.47%)	3,396	0.94	0.3460	0.88%	1,957	(1.43)	0.1522

The returns for channel breakout buys (Figure 3) show no statistical significance, and returns wander both positive and negative of the baseline return. The returns for breakout sells are more interesting. The longer period (100 day or greater) runs do show statistical significance, but the returns are again on the "wrong" side! These statistics show that buying new lows is a more profitable strategy than selling them, at least within this sample and this timeframe. There are both statistical and practical (economically significant) excess returns around these channel breakout sell points in the sample we examined.

Figure 4. Channel Breakout Sell Returns

	10 day breakout down				20 day breakout down				100 day breakout down				Yearly breakout down			
days	$\mu_X - \mu_B$	df	t-val	p-val	$\mu_X - \mu_B$	df	t-val	p-val	$\mu_X - \mu_B$	df	t-val	p-val	$\mu_X - \mu_B$	df	t-val	p-val
1	0.05%	7,727	(0.77)	0.4395	0.04%	6,321	(0.57)	0.5693	(0.00%)	2,030	0.02	0.9866	0.02%	835	(0.08)	0.9353
2	0.13%	7,420	(1.51)	0.1302	0.08%	5,833	(0.85)	0.3957	0.22%	2,003	(1.39)	0.1649	0.36%	833	(1.31)	0.1907
3	0.20%	7,254	(2.01)	0.0441	0.06%	5,720	(0.57)	0.5693	0.23%	2,008	(1.29)	0.1959	0.24%	836	(0.78)	0.4349
4	0.26%	7,179	(2.35)	0.0189	0.09%	5,708	(0.74)	0.4612	0.36%	1,999	(1.78)	0.0749	0.35%	833	(1.02)	0.3088
5	0.32%	6,931	(2.57)	0.0102	0.09%	5,608	(0.68)	0.4957	0.57%	2,020	(2.71)	0.0068	0.69%	836	(1.90)	0.0578
10	0.30%	7,065	(1.86)	0.0623	0.03%	5,787	(0.19)	0.8460	0.83%	2,021	(3.03)	0.0025	1.29%	838	(2.80)	0.0052
15	0.21%	6,994	(1.06)	0.2881	0.15%	5,672	(0.73)	0.4648	1.27%	2,034	(3.94)	0.0001	1.23%	842	(2.33)	0.0200
20	0.31%	6,740	(1.36)	0.1736	0.22%	5,570	(0.92)	0.3596	1.21%	2,015	(3.25)	0.0012	2.13%	836	(3.34)	0.0009
25	0.24%	6,776	(0.96)	0.3358	0.34%	5,551	(1.32)	0.1875	1.24%	2,001	(2.94)	0.0033	2.15%	833	(2.94)	0.0034
30	0.43%	6,862	(1.59)	0.1128	0.52%	5,557	(1.80)	0.0712	1.40%	1,995	(3.00)	0.0028	2.43%	831	(2.95)	0.0033
35	0.47%	6,844	(1.65)	0.0992	0.50%	5,494	(1.59)	0.1114	1.61%	2,003	(3.27)	0.0011	3.01%	832	(3.46)	0.0006
40	0.38%	6,995	(1.28)	0.2013	0.34%	5,625	(1.05)	0.2953	1.51%	2,036	(3.03)	0.0025	2.89%	840	(3.42)	0.0007
45	0.40%	6,950	(1.28)	0.2003	0.48%	5,580	(1.43)	0.1538	1.64%	2,012	(3.07)	0.0021	3.07%	836	(3.37)	0.0008
50	0.44%	6,913	(1.33)	0.1841	0.36%	5,578	(1.03)	0.3032	1.60%	2,035	(2.97)	0.0030	2.67%	843	(3.04)	0.0024
55	0.43%	6,932	(1.27)	0.2029	0.21%	5,657	(0.58)	0.5606	1.58%	1,962	(2.49)	0.0128	3.14%	828	(2.86)	0.0043
60	0.46%	6,854	(1.31)	0.1901	0.34%	5,554	(0.89)	0.3724	1.75%	2,013	(2.93)	0.0034	3.20%	838	(3.21)	0.0014
65	0.25%	6,858	(0.68)	0.4989	0.48%	5,512	(1.21)	0.2274	2.04%	2,019	(3.33)	0.0009	3.21%	842	(3.25)	0.0012
70	0.41%	6,841	(1.07)	0.2826	0.47%	5,519	(1.14)	0.2546	2.10%	2,026	(3.36)	0.0008	3.38%	842	(3.32)	0.0009
75	0.60%	6,809	(1.53)	0.1261	0.67%	5,543	(1.62)	0.1055	2.27%	2,031	(3.58)	0.0003	2.90%	846	(2.91)	0.0038
80	0.76%	6,778	(1.90)	0.0579	0.77%	5,567	(1.82)	0.0684	2.15%	2,059	(3.45)	0.0006	2.20%	850	(2.22)	0.0268
85	0.57%	6,898	(1.43)	0.1523	0.77%	5,536	(1.78)	0.0746	1.97%	2,068	(3.15)	0.0016	1.91%	851	(1.92)	0.0547
90	0.56%	6,889	(1.37)	0.1694	0.63%	5,571	(1.44)	0.1490	2.04%	2,066	(3.20)	0.0014	1.67%	852	(1.67)	0.0959
95	0.73%	6,859	(1.75)	0.0801	0.78%	5,524	(1.74)	0.0827	2.26%	2,065	(3.49)	0.0005	1.41%	852	(1.38)	0.1675
99	0.73%	6,902	(1.74)	0.0812	0.87%	5,522	(1.92)	0.0554	1.94%	2,087	(3.03)	0.0025	0.83%	855	(0.82)	0.4149

The two patterns examined so far were the patterns driving the systems that Brock, Lakonishok and LeBaron used in their paper. Without presenting detailed results, we discovered that the systems they used are no longer profitable (and never were profitable on many financial instruments other than the one index they examined), and we have shown here that the basic patterns initiating trades in these systems do not appear to have a positive expectation. In addition, our own trading experience has led us to believe that "failed breakouts are more common than momentum breakouts", meaning that we have made many profitable trades (and a net profit) by fading breakouts, or by buying new lows in prices. We also briefly examined other time periods (1980's and mid 1990's) and found that there was more of a positive expectation to the breakout strategies in those time periods. This could be concrete evidence of the market evolving and assimilating the technical trading activity of market participants, thus making those strategies no longer profitable. We simply found no edge to moving average crossovers, and believe that any edge attributed to them in the literature may be the result of curve fitting and over-optimization.

We also ran preliminary tests on a number of other technical patterns (stochastic crossovers, MACD divergence and crossing trades, several other parameter sets to the moving average crossover, and sophisticated digital signal filter based indicators) and found no patterns that were statistically significant. While this does not disprove conventional technical analysis, it does suggest that the patterns common in books, magazines, and journalists' commentary may not have a statistically verifiable edge.

[redacted]

Since we were looking for market anomalies and potential inefficiencies, we chose to examine price patterns that were in themselves anomalies. We have seen thousands of publicly available trading systems, and the only one that tests profitably across a sufficiently wide range of markets and parameters is Aberration designed by Ken Fitschen in 1986[2]. This system identifies large standard deviation price moves and takes positions against the moves.

[redacted] We found similar results with all methods but chose the percent [redacted] method as being the most simple and direct measure.

Figure 5 [redacted]

	[redacted]											
days	$\mu_X - \mu_B$	df	t-val	p-val	$\mu_X - \mu_B$	df	t-val	p-val	$\mu_X - \mu_B$	df	t-val	p-val
1	(0.09%)	13,392	1.74	0.0819	(0.10%)	5,476	1.20	0.2287	(0.22%)	1,859	1.43	0.1537
2	(0.12%)	11,999	1.72	0.0857	(0.16%)	5,375	1.57	0.1170	(0.35%)	1,864	1.95	0.0509
3	(0.19%)	11,916	2.29	0.0221	(0.10%)	5,318	0.90	0.3706	(0.35%)	1,864	1.68	0.0933
4	(0.21%)	11,578	2.33	0.0200	(0.11%)	5,227	0.85	0.3943	(0.34%)	1,856	1.45	0.1469
5	(0.18%)	11,152	1.72	0.0852	(0.10%)	5,201	0.70	0.4849	(0.08%)	1,850	0.32	0.7491
10	(0.28%)	10,066	1.94	0.0524	0.01%	4,981	(0.06)	0.9522	(0.15%)	1,831	0.41	0.6790
15	(0.32%)	10,118	1.88	0.0601	0.30%	5,022	(1.25)	0.2098	0.51%	1,835	(1.20)	0.2306
20	(0.65%)	9,929	3.33	0.0009	0.29%	4,975	(1.07)	0.2851	0.68%	1,835	(1.44)	0.1512
25	(0.83%)	9,757	3.78	0.0002	0.07%	5,000	(0.22)	0.8220	0.72%	1,832	(1.36)	0.1749
30	(0.87%)	9,801	3.59	0.0003	0.09%	5,019	(0.27)	0.7866	1.19%	1,835	(2.07)	0.0386
35	(0.93%)	9,854	3.63	0.0003	0.30%	5,019	(0.88)	0.3814	1.37%	1,839	(2.26)	0.0240
40	(0.98%)	9,976	3.59	0.0003	0.24%	5,074	(0.68)	0.4996	1.60%	1,831	(2.41)	0.0159
45	(0.71%)	9,669	2.48	0.0130	0.19%	5,032	(0.50)	0.6159	1.52%	1,834	(2.23)	0.0257
50	(0.81%)	9,741	2.72	0.0066	0.06%	5,060	(0.15)	0.8832	1.51%	1,828	(2.09)	0.0368
55	(0.84%)	9,784	2.70	0.0069	0.39%	5,056	(0.95)	0.3430	1.99%	1,819	(2.57)	0.0101
60	(0.93%)	9,752	2.91	0.0036	0.34%	5,075	(0.82)	0.4137	2.03%	1,827	(2.61)	0.0092
65	(1.02%)	9,846	3.10	0.0020	0.28%	5,096	(0.65)	0.5156	1.95%	1,834	(2.48)	0.0132
70	(0.77%)	9,901	2.29	0.0219	0.16%	5,109	(0.36)	0.7172	1.85%	1,836	(2.29)	0.0224
75	(0.75%)	9,938	2.20	0.0281	0.08%	5,058	(0.18)	0.8604	1.56%	1,831	(1.86)	0.0636
80	(0.42%)	9,811	1.19	0.2344	0.31%	5,079	(0.66)	0.5068	1.06%	1,838	(1.26)	0.2089
85	(0.26%)	9,779	0.72	0.4685	0.55%	5,079	(1.16)	0.2468	1.52%	1,834	(1.76)	0.0780
90	(0.11%)	9,773	0.30	0.7659	0.56%	5,064	(1.15)	0.2509	1.70%	1,831	(1.91)	0.0558
95	(0.02%)	9,787	0.06	0.9501	0.49%	5,042	(0.99)	0.3205	1.32%	1,833	(1.47)	0.1418
99	(0.14%)	9,860	0.37	0.7081	0.41%	5,032	(0.81)	0.4193	1.07%	1,833	(1.18)	0.2385

[2] http://www.trade-system.com/aberration.html

We first examined [redacted]. (See Figure 5 for results for 1%, 5% and 10% moves.) Our assumption was that a 1% move would not be significant since 1% moves are not uncommon in stocks. We were surprised to see statistically significance evidence of mean-reversion after 1% [redacted] and it was interesting to note that the effect only persisted approximately 60 trading days after the event. 5% [redacted] showed no statistical significance, but it is interesting to note that the volatility (measured by standard deviation of returns) is quite a bit higher than the baseline after the event. From a visual examination of scatterplots and time series charts of the data, it appears that larger moves are common after 5% [redacted] but that there is no directional bias. This condition would not be captured in our t-tests and further research is needed. 10% [redacted] quite unusual in the market, and there appears to be both statistically and practically significant anomalous returns following these events. There is a tendency for mean reversion following the event (sub-baseline returns), with continued movement (momentum?) in the direction of the initial large move for some time following the event. We also found this tendency began to weaken 60 trading days after the event as the mean returns drifted toward the baseline once again.

Figure 6. [redacted]

	[redacted]											
days	$\mu_X - \mu_B$	df	t-val	p-val	$\mu_X - \mu_B$	df	t-val	p-val	$\mu_X - \mu_B$	df	t-val	p-val
1	0.02%	10,561	(0.25)	0.8033	0.42%	4,396	(4.45)	0.0000	1.24%	1,277	(4.56)	0.0000
2	0.01%	10,667	(0.14)	0.8893	0.58%	4,228	(4.79)	0.0000	1.33%	1,280	(4.21)	0.0000
3	(0.09%)	10,541	1.00	0.3153	0.48%	4,323	(3.62)	0.0003	1.37%	1,278	(3.68)	0.0002
4	(0.13%)	10,602	1.29	0.1978	0.60%	4,261	(3.93)	0.0001	1.33%	1,285	(3.50)	0.0005
5	(0.09%)	10,683	0.81	0.4204	0.62%	4,304	(3.79)	0.0002	1.54%	1,288	(3.84)	0.0001
10	0.01%	10,864	(0.09)	0.9259	1.08%	4,169	(4.79)	0.0000	1.99%	1,299	(4.13)	0.0000
15	(0.04%)	10,806	0.21	0.8329	1.13%	4,199	(4.24)	0.0000	2.40%	1,312	(4.60)	0.0000
20	0.08%	10,633	(0.40)	0.6855	1.37%	4,169	(4.51)	0.0000	3.00%	1,307	(4.95)	0.0000
25	0.21%	10,496	(0.97)	0.3297	1.41%	4,194	(4.24)	0.0000	3.24%	1,309	(4.90)	0.0000
30	0.36%	10,516	(1.53)	0.1257	1.40%	4,171	(3.80)	0.0001	3.17%	1,314	(4.50)	0.0000
35	0.40%	10,400	(1.59)	0.1113	1.75%	4,217	(4.55)	0.0000	3.95%	1,314	(5.26)	0.0000
40	0.28%	10,779	(1.06)	0.2902	1.79%	4,221	(4.36)	0.0000	4.32%	1,316	(5.41)	0.0000
45	0.39%	10,656	(1.42)	0.1570	1.76%	4,203	(4.12)	0.0000	4.11%	1,313	(4.92)	0.0000
50	0.49%	10,735	(1.72)	0.0851	1.94%	4,219	(4.38)	0.0000	4.25%	1,314	(4.90)	0.0000
55	0.44%	10,690	(1.48)	0.1391	1.71%	4,256	(3.77)	0.0002	4.08%	1,305	(4.27)	0.0000
60	0.47%	10,605	(1.54)	0.1244	1.75%	4,220	(3.68)	0.0002	4.17%	1,308	(4.31)	0.0000
65	0.27%	10,569	(0.86)	0.3884	1.77%	4,225	(3.61)	0.0003	5.09%	1,306	(5.01)	0.0000
70	0.28%	10,618	(0.86)	0.3886	1.81%	4,217	(3.55)	0.0004	5.03%	1,310	(4.93)	0.0000
75	0.26%	10,718	(0.77)	0.4403	2.12%	4,225	(4.10)	0.0000	5.49%	1,310	(5.28)	0.0000
80	0.42%	10,688	(1.24)	0.2156	2.22%	4,234	(4.20)	0.0000	5.44%	1,312	(5.16)	0.0000
85	0.18%	10,617	(0.53)	0.5960	2.23%	4,216	(4.13)	0.0000	5.52%	1,314	(5.25)	0.0000
90	0.12%	10,568	(0.34)	0.7337	2.19%	4,225	(3.99)	0.0001	5.55%	1,315	(5.18)	0.0000
95	0.16%	10,580	(0.43)	0.6645	2.37%	4,177	(4.16)	0.0000	5.20%	1,318	(4.88)	0.0000
99	0.12%	10,635	(0.33)	0.7414	2.28%	4,198	(3.98)	0.0001	5.52%	1,316	(5.04)	0.0000

[redacted] moves are not a mirror image of [redacted] moves. The 1% price moves showed no statistical tendency whatsoever. However, larger moves (5% and 10%) showed remarkable excess returns and associated p-values. We were surprised by these results and found them to be remarkable evidence suggestive of market inefficiencies. We also investigated these moves on other samples of stocks and found that the tendency exists to varying degrees in other samples as well. Again, more research is needed.

Figure 7. [redacted]

	[redacted]							
days	$\mu_X - \mu_B$	df	t-val	p-val	$\mu_X - \mu_B$	df	t-val	p-val
1	(0.73%)	1,532	3.61	0.0003	(1.73%)	429	3.09	0.0021
2	(0.74%)	1,547	3.26	0.0011	(1.60%)	430	2.74	0.0065
3	(0.62%)	1,549	2.41	0.0163	(1.47%)	431	2.25	0.0250
4	(0.24%)	1,558	0.90	0.3704	(1.22%)	431	1.80	0.0720
5	(0.16%)	1,558	0.52	0.6003	(1.21%)	432	1.81	0.0710
10	0.21%	1,572	(0.56)	0.5763	(0.20%)	432	0.23	0.8157
15	0.52%	1,581	(1.23)	0.2201	0.10%	434	(0.11)	0.9158
20	0.91%	1,575	(1.87)	0.0618	0.10%	434	(0.10)	0.9237
25	1.15%	1,575	(2.13)	0.0334	(0.20%)	434	0.17	0.8642
30	1.59%	1,560	(2.54)	0.0113	1.08%	433	(0.78)	0.4342
35	1.80%	1,563	(2.71)	0.0069	1.24%	433	(0.87)	0.3842
40	1.83%	1,573	(2.67)	0.0076	0.72%	433	(0.47)	0.6354
45	2.06%	1,554	(2.72)	0.0067	0.55%	434	(0.37)	0.7116
50	2.08%	1,565	(2.73)	0.0064	0.26%	434	(0.17)	0.8663
55	2.05%	1,568	(2.63)	0.0086	0.02%	435	(0.01)	0.9917
60	1.82%	1,569	(2.28)	0.0227	0.60%	435	(0.37)	0.7115
65	1.85%	1,570	(2.23)	0.0257	0.37%	435	(0.22)	0.8236
70	2.07%	1,571	(2.44)	0.0149	0.48%	435	(0.29)	0.7747
75	1.69%	1,573	(1.96)	0.0504	0.22%	435	(0.13)	0.8966
80	1.61%	1,570	(1.80)	0.0718	0.88%	435	(0.50)	0.6163
85	2.04%	1,559	(2.16)	0.0310	0.70%	435	(0.40)	0.6918
90	1.66%	1,563	(1.74)	0.0812	0.57%	436	(0.33)	0.7430
95	1.38%	1,569	(1.47)	0.1418	0.23%	436	(0.13)	0.8953
99	1.24%	1,572	(1.32)	0.1884	(0.70%)	436	0.40	0.6894

We then examined large [redacted]. Conceptually, what is happening here is that something happens [redacted] We expected this should create imbalance since many traders will be trapped on the wrong side of the news, many will have large profits and be quick to sell, and the financial journalists will at least be reporting the event throughout the day so we can expect increased public awareness and interest. We expected to find imbalances and anomalous returns around these events. Figure 7 shows the results of two of our tests. To generalize, there appears to be a strong tendency for the market to close down in the days following [redacted] The magnitude of the down move is positively correlated to the size of the [redacted]. This tendency is very short term—most of our tests showed significant degradation after 4 trading days. There appears to be a second tendency for positive returns in the 30-60 days following the event, but this is less significant. [redacted] are very infrequent so our tests on that pattern suffered from small sample sizes. We suggest that the 10% moves up in Figure 7 do not show significant p-values because of the small sample size and plan to continue the research with a larger sample.

Figure 8. [redacted]

	[redacted]							
days	$\mu_X - \mu_B$	df	t-val	p-val	$\mu_X - \mu_B$	df	t-val	p-val
1	1.71%	1,448	(7.17)	0.0000	3.02%	456	(4.39)	0.0000
2	1.51%	1,457	(5.60)	0.0000	2.88%	457	(3.95)	0.0001
3	1.34%	1,461	(4.49)	0.0000	3.13%	458	(4.09)	0.0001
4	1.43%	1,462	(4.35)	0.0000	2.76%	458	(3.37)	0.0008
5	1.76%	1,466	(5.01)	0.0000	2.52%	458	(3.06)	0.0023
10	2.45%	1,483	(5.91)	0.0000	3.16%	460	(3.35)	0.0009
15	2.48%	1,491	(5.19)	0.0000	2.76%	461	(2.68)	0.0077
20	2.65%	1,497	(5.08)	0.0000	3.15%	461	(2.74)	0.0065
25	2.57%	1,500	(4.50)	0.0000	3.35%	462	(2.81)	0.0052
30	2.59%	1,505	(4.23)	0.0000	4.06%	462	(3.15)	0.0017
35	2.61%	1,502	(3.93)	0.0001	3.76%	462	(2.72)	0.0067
40	2.76%	1,502	(3.89)	0.0001	4.15%	462	(2.85)	0.0046
45	3.47%	1,495	(4.60)	0.0000	6.03%	461	(3.65)	0.0003
50	3.39%	1,493	(4.27)	0.0000	4.87%	462	(2.94)	0.0034
55	3.60%	1,499	(4.47)	0.0000	6.67%	458	(2.81)	0.0052
60	4.29%	1,497	(5.14)	0.0000	6.97%	460	(3.45)	0.0006
65	4.12%	1,499	(4.82)	0.0000	6.29%	462	(3.43)	0.0007
70	4.15%	1,494	(4.59)	0.0000	6.04%	461	(3.10)	0.0020
75	4.69%	1,494	(5.06)	0.0000	6.35%	461	(3.18)	0.0016
80	5.12%	1,492	(5.36)	0.0000	7.68%	461	(3.66)	0.0003
85	5.02%	1,494	(5.23)	0.0000	8.24%	461	(3.86)	0.0001
90	4.92%	1,499	(5.15)	0.0000	7.97%	461	(3.76)	0.0002
95	4.54%	1,501	(4.72)	0.0000	7.56%	462	(3.65)	0.0003
99	3.99%	1,501	(4.09)	0.0000	6.71%	462	(3.24)	0.0013

Again, buying equities does not appear to be the mirror image of selling. We found a stronger and simpler tendency on the downside: [redacted] the market shows a very strong tendency to immediately trade back up and this tendency continues for the 100 day length of our test window. We were surprised by the magnitude of the effect and also by the statistical significance. (One of our tests showed t-values between 6 and 8.) Obviously, further research is warranted.

We repeat again that we are reasonably certain these effects are not the result of anomalies in our sample. We repeated much of this work with different samples, different sample sizes, and different time periods. We found that the effects were not stable across time and have been stronger in some time periods than others. We also found that not all samples of stocks exhibit the effects to the same degree, but we did find these unusual returns in all samples we examined.

Efficient Market Models

Overview of the Theory

The efficient market hypothesis (EMH) is the basis for most of modern academic research into the theory of stock movements, and accepted as gospel by much of the research community, albeit there is an academic debate as to whether or not the market is weak-form, semi-strong or strong form efficient. These theories have each been extensively researched, and a large amount of statistical evidence supports that there is a strong basis for these models. However, many portfolio managers, traders and hedge funds still rely on technical tools to help in their trading strategies, and many are able to outperform the market on a regular basis. Academics dismiss this ability to outperform as a statistical outlier, pointing out that at least a few people are going to randomly outperform, given the large pool of people who either peer perform or under perform the market.

The most generally accepted form of efficient market theory, and the model which we are testing, is the weak-form efficient theory. The weak-form efficiency theory is based on the following assumptions:

- The current share price is the best and unbiased estimate of the intrinsic value of the security.
- No excess returns can be made by analyzing the historical price.
- It does allow for the possibility of an analyst making excess returns by analyzing financial statements.
- It does allow for the possibility of insiders, with insider information, to make excess returns.

Semi-strong and Strong form theory include all of the assumptions that weak form theory does, in that no excess returns can be made off of just historical prices. The difference between the semi-strong, strong-form and the weak-form model is that the semi-strong form does not allow analysts to, on average, make excess returns. The strong form goes one step further and states that no one can make excess returns on an average basis.

From these assumptions, a multitude of models have been developed for the price patterns of the stock market. These models were used to compare the effectiveness of our trading rules against simulated data, thereby creating our own market which we can compare against the actual market. The models we studied were the same models mentioned in the work of Brock, Lakonishok and LeBaron (1992), which included an autoregressive model of order one (AR model), a generalized autoregressive conditional heteroskedasticity in-means model (GARCH-M) and an exponential version of the GARCH model (EGARCH). Each of these models have been shown to fit well with historical prices of market indexes, and were chosen because previous research has shown that the market has shown a deviation from normal random walk behavior and that the distribution of market returns is non-normal. Each of these models, to varying degrees, takes into account the non-normal distributions of returns and "corrects" the expected return to account for this.

The AR model is defined by the equation:

$$r_t = b + \rho \cdot r_{t-1} + \varepsilon_t$$

Where r_t is defined as the continuously compounded return on day t and ε_t is the independent and normally distributed error term. (Note that the error term is not standard normal, in that the standard deviation does not have to be 0. However, the mean of the error was required to be 0.) Terms b and ρ are determined from fitting the equation to a historical data set. This model builds in the one day autocorrelation that had previously been seen in the stock market by Conrad and Kaul (1990). Multiple day autocorrelation could have been sampled, however the results were previously shown to be statistically insignificant.

The second, and significantly more complex model, is the GARCH-M model, given by the equations:

$$r_t = a + \gamma \cdot h_t + b \cdot \varepsilon_{t-1} + \varepsilon_t$$
$$h_t = \alpha_0 + \alpha_1 \cdot \varepsilon_{t-1}^t + \beta \cdot h_{t-1}$$
$$\varepsilon_t = h_t^{1/2} z_t$$
$$z_t \sim N(0,1)$$

Where r_t is defined as the continuously compounded return on day t, and z is a standard normal distribution. The rest of the variables were fit using the historical time series of prices. This model is useful because it takes into account the historical volatility and the past returns. Under this process, the volatility is not assumed to be constant, but instead can change over time.

The final model used to test our trading systems was the E-GARCH model given by the equations:

$$r_t = a + \gamma \cdot e^{h_t} + b \cdot \varepsilon_{t-1} + \varepsilon_t$$
$$h_t = \alpha_0 + g(z_{t-1}) + \beta \cdot h_{t-1}$$
$$g(z_t) = \theta \cdot z_t + \omega(|z_t| - (2/\pi)^{1/2})$$
$$\varepsilon_t = e^{(1/2)h_t} z_t$$
$$z_t \sim N(0,1)$$

This model is very similar to the GARCH-M model, with two important differences. First, the conditional variance follows an autoregressive process and second, not only does the volatility affect future values differently, different signs on the volatility affect the future values in different ways. This gives the potential to capture the increased volatility after a down moves as seen by Black (1976).

Building the Models: AR, GARCH-M, EGARCH

AR Model

The AR model was the only model out of the three in which the constants were solved by fitting the data to historical data. The data was fit to the S&P 500 starting from December 1921. Using Mathematica, the data was imported in matrix form and stripped of all extraneous information except for daily returns. The matrix was then multiplied by ρ and b was added to each term (in variable format). Once the matrix was set up, the actual return for the next day was compared to the expected return, and an error term was calculated by subtracting the two. (The last term from the expected return matrix was dropped, and the first term from the actual return matrix was dropped, allowing us to subtract the two matrixes in one step.) Once each error term was found, they were then summed to give us a total error. To solve for ρ and b, we found the terms that minimized the total error.

Not surprisingly, the fit terms matched very closely with the work of Brock, Lakonishok and LeBaron (1992) and resulted in a being .000212 and ρ being 0.03569 (compared with their work of .00015 and .03330 respectively). In addition to the variables, We were able to also estimate the standard deviation of the error term (.0116213) and verify that the mean of the error was in fact 0 (-2.3×10^{-19}).

Once the parameters were verified, the model was then built using 100 tests with 10,000 days of data. Using a random number generator built into Excel, a normal distribution with a mean of 0 and the variance found above were added to each return. The expected returns were then converted into an estimated price for each day, starting with a base price of 100.

GARCH-M

While we would have also liked to verify the constants on the other two tests, we were unsuccessful at fitting the data using the programs at our disposal. The reason is that each term is dependent on the term before it, and to solve for each variable exactly would lead to over 19,000 variables which would have to be solved for. Further work should be done to verify Brock, Lakonishok and LeBaron's work, however for expediency and to use a more complex simulation of the data, we chose to use the estimates found in their paper. (See page 1747 of Brock, Lakonishok and LeBaron, 1992.)

The model was built by first creating a set of 100 by 10,000 random numbers distributed in a standard normal distribution. The model was then built up by solving for h (which is dependent on each h before it), and then solving for the returns which are dependent on the current h, the previous h, the current error and the previous error term.

Initial conditions proved to be very important within the model, and lacking the ability to fit the data, the initial conditions were "backed into" by making an initial assumption of h-not and running the simulation. The average h was then found after discarding the first 1000 samples in the time series, to mute any effects the initial condition had upon the data. The data was then re-run with the initial h-not set to the average that we just found. After doing this, we found the initial condition closely matched the average, and therefore did not tilt the data in any specific direction.

After building the GARCH-M data, we found that we had very large (excessively large) average returns throughout our data, and wonder if some of the constants, especially γ might be overly inflated. However, without the ability to fit the data ourselves, we continued on the assumption that the data in the article was correct.

EGARCH

The EGARCH model was built in much of the same way as the GARCH-M model. We began by first creating a set of 100 by 10,000 random numbers distributed in a standard normal distribution, and building up the model in Excel much the same way as we did in the GARCH-M test.

The EGARCH data, on examination, produced returns that were much closer inline with what we see in the real world over the same period. While this is not proof that there is something wrong with the GARCH-M or inherently better with the EGARCH model, it does bring up concern that there is something wrong with the way we modeled the GARCH-M data.

Bootstrap Tests

Using the data resulting from the models, we compared our baseline results with the simulated data. We chose to compare the data with [redacted] model and the Close [redacted] trading models because these models showed the most significance in the real data as described above. Further testing on the other trading models is needed and should be the subject of future research.

There was one abnormality noted in the simulated data that did impede our ability to compare the real data to the simulated data. The propensity for large moves (5% and 10% moves) were significantly less in the simulated data than the data from the actual market, and to do good comparisons significantly more data would have had to been simulated than our computing power allowed for. This may be a result of our assumption of the distribution of random data was normal, when a better distribution (with fatter tails) might have been more appropriate. Further research might include a look into the effect the distribution of error terms has on the underlying stock prices and whether or not the distribution of error terms follows a predictable pattern.

[redacted] Up Comparison

Market Data

1%

days	$\mu_X - \mu_B$	df	t-val	p-val
1	(0.09%)	13,392	1.74	0.0819
2	(0.12%)	11,999	1.72	0.0857
3	(0.19%)	11,916	2.29	0.0221
4	(0.21%)	11,578	2.33	0.0200
5	(0.18%)	11,152	1.72	0.0852
10	(0.28%)	10,066	1.94	0.0524
15	(0.32%)	10,118	1.88	0.0601
20	(0.65%)	9,929	3.33	0.0009
25	(0.83%)	9,757	3.78	0.0002
30	(0.87%)	9,801	3.59	0.0003
35	(0.93%)	9,854	3.63	0.0003
40	(0.98%)	9,976	3.59	0.0003
45	(0.71%)	9,669	2.48	0.0130
50	(0.81%)	9,741	2.72	0.0066
55	(0.84%)	9,784	2.70	0.0069
60	(0.93%)	9,752	2.91	0.0036
65	(1.02%)	9,846	3.10	0.0020
70	(0.77%)	9,901	2.29	0.0219
75	(0.75%)	9,938	2.20	0.0281
80	(0.42%)	9,811	1.19	0.2344
85	(0.26%)	9,779	0.72	0.4685
90	(0.11%)	9,773	0.30	0.7659
95	(0.02%)	9,787	0.06	0.9501
99	(0.14%)	9,860	0.37	0.7081

2.50%

days	$\mu_X - \mu_B$	df	t-val	p-val
1	(0.17%)	10,013	2.78	0.0054
2	(0.20%)	9,575	2.56	0.0106
3	(0.19%)	9,372	2.06	0.0392
4	(0.22%)	9,141	2.14	0.0326
5	(0.19%)	8,978	1.72	0.0858
10	(0.26%)	8,321	1.62	0.1051
15	(0.35%)	8,642	1.89	0.0593
20	(0.49%)	8,436	2.31	0.0212
25	(0.62%)	8,533	2.68	0.0074
30	(0.50%)	8,521	1.97	0.0485
35	(0.35%)	8,450	1.27	0.2038
40	(0.30%)	8,464	1.01	0.3109
45	(0.10%)	8,307	0.34	0.7337
50	(0.21%)	8,297	0.67	0.5053
55	(0.28%)	8,312	0.83	0.4051
60	(0.27%)	8,306	0.78	0.4327
65	(0.28%)	8,346	0.80	0.4220
70	(0.16%)	8,355	0.43	0.6652
75	(0.12%)	8,365	0.33	0.7384
80	(0.01%)	8,367	0.04	0.9711
85	0.09%	8,346	(0.24)	0.8087
90	0.11%	8,390	(0.29)	0.7719
95	0.03%	8,407	(0.07)	0.9423
99	(0.02%)	8,444	0.04	0.9677

5%

days	$\mu_X - \mu_B$	df	t-val	p-val
1	(0.10%)	5,476	1.20	0.2287
2	(0.16%)	5,375	1.57	0.1170
3	(0.10%)	5,318	0.90	0.3706
4	(0.11%)	5,227	0.85	0.3943
5	(0.10%)	5,201	0.70	0.4849
10	0.01%	4,981	(0.06)	0.9522
15	0.30%	5,022	(1.25)	0.2098
20	0.29%	4,975	(1.07)	0.2851
25	0.07%	5,000	(0.22)	0.8220
30	0.09%	5,019	(0.27)	0.7866
35	0.30%	5,019	(0.88)	0.3814
40	0.24%	5,074	(0.68)	0.4996
45	0.19%	5,032	(0.50)	0.6159
50	0.06%	5,060	(0.15)	0.8832
55	0.39%	5,056	(0.95)	0.3430
60	0.34%	5,075	(0.82)	0.4137
65	0.28%	5,096	(0.65)	0.5156
70	0.16%	5,109	(0.36)	0.7172
75	0.08%	5,058	(0.18)	0.8604
80	0.31%	5,079	(0.66)	0.5068
85	0.55%	5,079	(1.16)	0.2458
90	0.56%	5,064	(1.15)	0.2509
95	0.49%	5,042	(0.99)	0.3205
99	0.41%	5,032	(0.81)	0.4193

10%

days	$\mu_X - \mu_B$	df	t-val	p-val
1	(0.22%)	1,859	1.43	0.1537
2	(0.35%)	1,864	1.95	0.0509
3	(0.35%)	1,864	1.68	0.0933
4	(0.34%)	1,856	1.45	0.1459
5	(0.08%)	1,850	0.32	0.7491
10	(0.15%)	1,831	0.41	0.6790
15	0.51%	1,835	(1.20)	0.2306
20	0.68%	1,835	(1.44)	0.1512
25	0.72%	1,832	(1.36)	0.1749
30	1.19%	1,835	(2.07)	0.0386
35	1.37%	1,839	(2.26)	0.0240
40	1.60%	1,831	(2.41)	0.0159
45	1.52%	1,834	(2.23)	0.0257
50	1.51%	1,828	(2.09)	0.0368
55	1.99%	1,819	(2.57)	0.0101
60	2.03%	1,827	(2.61)	0.0092
65	1.95%	1,834	(2.48)	0.0132
70	1.85%	1,836	(2.29)	0.0224
75	1.56%	1,831	(1.86)	0.0636
80	1.06%	1,838	(1.26)	0.2089
85	1.52%	1,834	(1.76)	0.0780
90	1.70%	1,831	(1.91)	0.0558
95	1.32%	1,833	(1.47)	0.1418
99	1.07%	1,833	(1.18)	0.2385

AR Data

1% AR

days	$\mu_X - \mu_B$	df	t-val	p-val
1	0.04%	18,716	(2.79)	0.0053
2	0.03%	18,818	(1.38)	0.1684
3	0.03%	18,720	(1.17)	0.2404
4	0.03%	18,649	(1.08)	0.2789
5	0.05%	18,465	(1.27)	0.2039
10	(0.06%)	18,450	1.22	0.2225
15	(0.07%)	18,344	1.19	0.2355
20	(0.17%)	18,291	2.36	0.0183
25	(0.27%)	18,343	3.43	0.0006
30	(0.35%)	18,270	4.03	0.0001
35	(0.44%)	18,209	4.80	0.0000
40	(0.45%)	18,231	4.65	0.0000
45	(0.54%)	18,221	5.30	0.0000
50	(0.68%)	18,100	6.41	0.0000
55	(0.75%)	17,978	6.81	0.0000
60	(0.79%)	17,836	6.91	0.0000
65	(0.84%)	17,905	7.15	0.0000
70	(0.92%)	17,827	7.63	0.0000
75	(0.94%)	17,726	7.66	0.0000
80	(1.03%)	17,672	8.19	0.0000
85	(1.04%)	17,552	8.13	0.0000
90	(1.11%)	17,410	8.50	0.0000
95	(1.18%)	17,374	8.88	0.0000
99	(1.20%)	17,185	8.94	0.0000

2.5% AR

days	$\mu_X - \mu_B$	df	t-val	p-val
1	0.09%	10,765	(4.89)	0.0000
2	0.07%	10,866	(2.79)	0.0053
3	0.07%	10,946	(2.20)	0.0278
4	0.08%	10,847	(2.22)	0.0267
5	0.11%	10,782	(2.60)	0.0094
10	0.03%	10,817	(0.53)	0.5988
15	(0.09%)	10,825	1.26	0.2075
20	(0.20%)	10,816	2.48	0.0131
25	(0.26%)	10,857	2.91	0.0036
30	(0.32%)	10,683	3.23	0.0012
35	(0.37%)	10,665	3.58	0.0003
40	(0.40%)	10,671	3.62	0.0003
45	(0.44%)	10,628	3.75	0.0002
50	(0.53%)	10,624	4.42	0.0000
55	(0.65%)	10,567	5.19	0.0000
60	(0.66%)	10,493	5.09	0.0000
65	(0.69%)	10,436	5.15	0.0000
70	(0.72%)	10,374	5.20	0.0000
75	(0.78%)	10,316	5.49	0.0000
80	(0.84%)	10,282	5.86	0.0000
85	(0.94%)	10,217	6.37	0.0000
90	(0.93%)	10,124	6.18	0.0000
95	(0.97%)	10,108	6.40	0.0000
99	(0.99%)	10,025	6.42	0.0000

5% AR

days	$\mu_X - \mu_B$	df	t-val	p-val
1	0.47%	24	(1.95)	0.0628
2	0.23%	24	(0.69)	0.4966
3	0.85%	24	(2.11)	0.0452
4	0.63%	24	(1.32)	0.2004
5	0.38%	24	(0.69)	0.4987
10	0.33%	24	(0.45)	0.6539
15	(0.78%)	24	1.01	0.3230
20	(1.01%)	24	1.33	0.1960
25	(0.36%)	24	0.45	0.6567
30	(0.76%)	24	0.84	0.4087
35	(1.00%)	24	1.01	0.3225
40	(1.00%)	24	1.02	0.3159
45	(1.11%)	24	0.96	0.3476
50	(2.12%)	24	1.79	0.0855
55	(2.14%)	24	1.72	0.0981
60	(1.27%)	24	0.95	0.3526
65	(1.60%)	24	1.22	0.2355
70	(1.78%)	24	1.27	0.2155
75	(2.56%)	24	1.79	0.0857
80	(2.77%)	24	1.78	0.0875
85	(4.22%)	24	2.99	0.0063
90	(4.09%)	24	2.78	0.0105
95	(2.91%)	24	2.09	0.0475
99	(2.69%)	24	1.99	0.0582

GARCH-M Data

1% GARCH-M

days	$\mu_X - \mu_B$	df	t-val	p-val
1	0.14%	17,515	(9.14)	0.0000
2	0.13%	17,527	(5.75)	0.0000
3	0.13%	17,602	(4.80)	0.0000
4	0.13%	17,532	(4.12)	0.0000
5	0.12%	17,563	(3.19)	0.0014
10	0.03%	17,416	(0.57)	0.5696
15	(0.01%)	17,223	0.16	0.8716
20	(0.13%)	17,363	1.81	0.0698
25	(0.18%)	17,306	2.27	0.0234
30	(0.25%)	17,248	2.81	0.0050
35	(0.19%)	17,223	1.97	0.0488
40	(0.28%)	17,322	2.78	0.0055
45	(0.37%)	17,231	3.53	0.0004
50	(0.48%)	17,261	4.33	0.0000
55	(0.55%)	17,164	4.77	0.0000
60	(0.59%)	17,018	4.96	0.0000
65	(0.69%)	17,050	5.56	0.0000
70	(0.77%)	17,089	6.06	0.0000
75	(0.86%)	16,962	6.62	0.0000
80	(1.00%)	16,956	7.48	0.0000
85	(1.11%)	16,911	8.14	0.0000
90	(1.19%)	16,883	8.52	0.0000
95	(1.29%)	16,656	9.04	0.0000
99	(1.36%)	16,692	9.45	0.0000

2.5% GARCH-M

days	$\mu_X - \mu_B$	df	t-val	p-val
1	0.28%	4,731	(10.52)	0.0000
2	0.28%	4,706	(7.08)	0.0000
3	0.33%	4,713	(6.60)	0.0000
4	0.37%	4,678	(6.30)	0.0000
5	0.35%	4,690	(5.46)	0.0000
10	0.41%	4,716	(4.45)	0.0000
15	0.58%	4,707	(5.24)	0.0000
20	0.58%	4,736	(4.59)	0.0000
25	0.57%	4,739	(4.06)	0.0001
30	0.62%	4,688	(4.00)	0.0001
35	0.66%	4,695	(3.97)	0.0001
40	0.64%	4,727	(3.64)	0.0003
45	0.64%	4,731	(3.49)	0.0005
50	0.64%	4,739	(3.35)	0.0008
55	0.61%	4,758	(3.12)	0.0018
60	0.65%	4,757	(3.20)	0.0014
65	0.62%	4,751	(2.95)	0.0032
70	0.58%	4,754	(2.69)	0.0072
75	0.61%	4,745	(2.78)	0.0055
80	0.61%	4,732	(2.66)	0.0078
85	0.61%	4,749	(2.65)	0.0081
90	0.42%	4,743	(1.79)	0.0732
95	0.42%	4,738	(1.76)	0.0778
99	0.39%	4,732	(1.60)	0.1097

5% GARCH-M

days	$\mu_X - \mu_B$	df	t-val	p-val
1	0.69%	516	(5.80)	0.0000
2	0.97%	516	(5.59)	0.0000
3	1.26%	515	(5.14)	0.0000
4	1.43%	515	(4.99)	0.0000
5	1.67%	514	(5.07)	0.0000
10	2.40%	515	(5.25)	0.0000
15	2.89%	515	(5.40)	0.0000
20	3.22%	516	(5.56)	0.0000
25	4.01%	515	(6.14)	0.0000
30	4.95%	515	(6.99)	0.0000
35	5.56%	515	(7.37)	0.0000
40	5.71%	515	(7.10)	0.0000
45	6.33%	515	(7.42)	0.0000
50	6.96%	515	(7.64)	0.0000
55	7.35%	515	(7.92)	0.0000
60	7.77%	515	(8.18)	0.0000
65	7.86%	515	(7.83)	0.0000
70	8.11%	515	(8.01)	0.0000
75	8.62%	515	(8.21)	0.0000
80	9.04%	515	(8.40)	0.0000
85	9.56%	515	(8.50)	0.0000
90	9.97%	515	(8.60)	0.0000
95	10.01%	515	(8.37)	0.0000
99	10.65%	515	(8.71)	0.0000

10% GARCH-M

days	$\mu_X - \mu_B$	df	t-val	p-val
1	1.49%	33	(1.81)	0.0793
2	2.31%	33	(2.04)	0.0495
3	4.23%	33	(2.31)	0.0272
4	4.66%	33	(2.04)	0.0494
5	3.77%	33	(1.48)	0.1476
10	9.83%	33	(3.12)	0.0037
15	10.79%	33	(2.74)	0.0098
20	12.33%	33	(2.69)	0.0111
25	14.11%	33	(2.88)	0.0069
30	21.27%	33	(3.77)	0.0006
35	22.51%	33	(3.78)	0.0006
40	20.30%	33	(3.38)	0.0019
45	20.91%	33	(3.67)	0.0009
50	25.20%	33	(3.40)	0.0018
55	27.09%	33	(3.22)	0.0028
60	29.39%	33	(3.17)	0.0033
65	27.88%	33	(3.58)	0.0011
70	28.32%	33	(3.55)	0.0012
75	30.27%	33	(3.63)	0.0009
80	29.41%	33	(3.62)	0.0010
85	31.82%	33	(3.57)	0.0011
90	33.45%	33	(3.66)	0.0009
95	35.24%	33	(3.47)	0.0015
99	37.62%	33	(3.25)	0.0027

EGARCH Data

1% EGARCH

days	$\mu_X - \mu_B$	df	t-val	p-val
1	0.13%	18,311	(8.23)	0.0000
2	0.11%	18,444	(4.89)	0.0000
3	0.11%	18,527	(3.94)	0.0001
4	0.13%	18,541	(3.75)	0.0002
5	0.12%	18,366	(3.06)	0.0022
10	0.01%	18,345	(0.12)	0.9067
15	(0.03%)	18,132	0.48	0.6331
20	(0.12%)	18,155	1.63	0.1033
25	(0.22%)	18,047	2.78	0.0054
30	(0.26%)	17,865	2.99	0.0028
35	(0.35%)	17,886	3.80	0.0001
40	(0.44%)	17,902	4.56	0.0000
45	(0.56%)	17,650	5.48	0.0000
50	(0.63%)	17,737	5.98	0.0000
55	(0.79%)	17,625	7.21	0.0000
60	(0.90%)	17,436	7.92	0.0000
65	(0.99%)	17,240	8.48	0.0000
70	(1.10%)	17,015	9.14	0.0000
75	(1.15%)	16,830	9.36	0.0000
80	(1.22%)	16,774	9.65	0.0000
85	(1.31%)	16,663	10.22	0.0000
90	(1.46%)	16,561	11.19	0.0000
95	(1.60%)	16,395	12.11	0.0000
99	(1.66%)	16,382	12.38	0.0000

2.5% EGARCH

days	$\mu_X - \mu_B$	df	t-val	p-val
1	0.30%	7,445	(13.52)	0.0000
2	0.29%	7,391	(8.76)	0.0000
3	0.29%	7,468	(6.96)	0.0000
4	0.28%	7,459	(5.93)	0.0000
5	0.30%	7,516	(5.74)	0.0000
10	0.32%	7,450	(4.35)	0.0000
15	0.42%	7,441	(4.64)	0.0000
20	0.42%	7,503	(4.16)	0.0000
25	0.38%	7,502	(3.44)	0.0006
30	0.47%	7,466	(3.93)	0.0001
35	0.33%	7,442	(2.60)	0.0092
40	0.32%	7,485	(2.42)	0.0157
45	0.20%	7,423	(1.44)	0.1507
50	0.17%	7,409	(1.13)	0.2603
55	0.04%	7,380	(0.24)	0.8093
60	(0.06%)	7,360	0.36	0.7218
65	(0.12%)	7,322	0.76	0.4480
70	(0.26%)	7,294	1.59	0.1125
75	(0.34%)	7,287	2.03	0.0425
80	(0.39%)	7,286	2.30	0.0214
85	(0.45%)	7,239	2.56	0.0104
90	(0.56%)	7,207	3.18	0.0015
95	(0.65%)	7,165	3.62	0.0003
99	(0.74%)	7,176	4.07	0.0000

5% EGARCH

days	$\mu_X - \mu_B$	df	t-val	p-val
1	0.79%	596	(8.40)	0.0000
2	0.94%	597	(6.99)	0.0000
3	1.03%	597	(6.14)	0.0000
4	1.19%	598	(6.26)	0.0000
5	1.38%	599	(6.64)	0.0000
10	1.87%	600	(6.65)	0.0000
15	2.44%	600	(7.34)	0.0000
20	2.44%	602	(6.85)	0.0000
25	2.21%	601	(5.54)	0.0000
30	2.40%	602	(5.63)	0.0000
35	2.25%	602	(5.05)	0.0000
40	2.44%	602	(5.12)	0.0000
45	2.12%	602	(4.29)	0.0000
50	2.06%	601	(3.93)	0.0001
55	2.26%	602	(4.25)	0.0000
60	2.24%	602	(4.09)	0.0000
65	1.99%	601	(3.48)	0.0005
70	1.98%	600	(3.34)	0.0009
75	1.80%	600	(3.02)	0.0026
80	1.57%	600	(2.54)	0.0113
85	1.52%	600	(2.43)	0.0156
90	1.34%	600	(2.11)	0.0356
95	1.05%	600	(1.65)	0.0996
99	1.00%	600	(1.57)	0.1178

Notice that while very little significance was observed in the real market data (with some abnormalities in the 1% moves in the medium term time horizon), very strong correlations were measured (albeit in different time frames) in the various models. This suggests that the market is already discounting the expectation of further upward moves after [redacted] and that would be expected using the EMH model. In addition, we see that the GARCH-M model produces returns that are well in excess of the baseline model, which is not something that is observed in the real data. This makes us question the validity of the GARCH-M model for [redacted] and overemphasizes the effect [redacted] currently have on future stock prices. (This may be a result of not allowing downward [redacted] to have different effects of upward [redacted]

Of the three models, the EGARCH model most closely approximates what we see in the real data, especially in the long term where there is only a .75% difference between the expected out-performance (the EGARCH model still over estimates the out-performance).

[redacted] Down Comparison

Market Data

1%

days	$\mu_X - \mu_B$	df	t-val	p-val
1	0.02%	10,561	(0.25)	0.8033
2	0.01%	10,667	(0.14)	0.8893
3	(0.09%)	10,541	1.00	0.3153
4	(0.13%)	10,602	1.29	0.1978
5	(0.09%)	10,683	0.81	0.4204
10	0.01%	10,864	(0.09)	0.9259
15	(0.04%)	10,806	0.21	0.8329
20	0.08%	10,633	(0.40)	0.6855
25	0.21%	10,496	(0.97)	0.3297
30	0.36%	10,516	(1.53)	0.1257
35	0.40%	10,400	(1.59)	0.1113
40	0.28%	10,779	(1.06)	0.2902
45	0.39%	10,656	(1.42)	0.1570
50	0.49%	10,735	(1.72)	0.0851
55	0.44%	10,690	(1.48)	0.1391
60	0.47%	10,605	(1.54)	0.1244
65	0.27%	10,569	(0.86)	0.3884
70	0.28%	10,618	(0.86)	0.3886
75	0.26%	10,718	(0.77)	0.4403
80	0.42%	10,688	(1.24)	0.2156
85	0.18%	10,617	(0.53)	0.5960
90	0.12%	10,568	(0.34)	0.7337
95	0.16%	10,580	(0.43)	0.6645
99	0.12%	10,635	(0.33)	0.7414

2.50%

days	$\mu_X - \mu_B$	df	t-val	p-val
1	0.04%	10,764	(0.73)	0.4682
2	0.01%	9,391	(0.07)	0.9461
3	(0.09%)	8,925	0.98	0.3269
4	(0.04%)	8,673	0.41	0.6791
5	0.01%	8,551	(0.11)	0.9142
10	0.16%	8,293	(1.02)	0.3084
15	0.13%	8,228	(0.69)	0.4910
20	(0.11%)	8,060	0.51	0.6114
25	(0.19%)	8,189	0.82	0.4099
30	(0.06%)	8,135	0.21	0.8315
35	0.07%	7,828	(0.25)	0.8064
40	0.31%	7,844	(1.01)	0.3140
45	0.45%	7,711	(1.39)	0.1632
50	0.63%	7,735	(1.87)	0.0610
55	0.59%	7,924	(1.74)	0.0820
60	0.63%	7,861	(1.78)	0.0754
65	0.64%	7,842	(1.73)	0.0830
70	0.71%	7,922	(1.88)	0.0605
75	0.90%	7,921	(2.33)	0.0197
80	0.96%	7,945	(2.44)	0.0147
85	0.96%	7,905	(2.41)	0.0161
90	0.93%	7,970	(2.30)	0.0213
95	0.88%	7,969	(2.14)	0.0325
99	0.77%	7,981	(1.85)	0.0640

5%

days	$\mu_X - \mu_B$	df	t-val	p-val
1	0.42%	4,396	(4.45)	0.0000
2	0.58%	4,228	(4.79)	0.0000
3	0.48%	4,323	(3.62)	0.0003
4	0.60%	4,261	(3.93)	0.0001
5	0.62%	4,304	(3.79)	0.0002
10	1.08%	4,169	(4.79)	0.0000
15	1.13%	4,199	(4.24)	0.0000
20	1.37%	4,169	(4.51)	0.0000
25	1.41%	4,194	(4.24)	0.0000
30	1.40%	4,171	(3.80)	0.0001
35	1.75%	4,217	(4.55)	0.0000
40	1.79%	4,221	(4.36)	0.0000
45	1.76%	4,203	(4.12)	0.0000
50	1.94%	4,219	(4.38)	0.0000
55	1.71%	4,256	(3.77)	0.0002
60	1.75%	4,220	(3.68)	0.0002
65	1.77%	4,225	(3.61)	0.0003
70	1.81%	4,217	(3.55)	0.0004
75	2.12%	4,225	(4.10)	0.0000
80	2.22%	4,234	(4.20)	0.0000
85	2.23%	4,216	(4.13)	0.0000
90	2.19%	4,225	(3.99)	0.0001
95	2.37%	4,177	(4.16)	0.0000
99	2.28%	4,198	(3.98)	0.0001

10%

days	$\mu_X - \mu_B$	df	t-val	p-val
1	1.24%	1,277	(4.56)	0.0000
2	1.33%	1,280	(4.21)	0.0000
3	1.37%	1,278	(3.68)	0.0002
4	1.33%	1,285	(3.50)	0.0005
5	1.54%	1,288	(3.84)	0.0001
10	1.99%	1,299	(4.13)	0.0000
15	2.40%	1,312	(4.60)	0.0000
20	3.00%	1,307	(4.95)	0.0000
25	3.24%	1,309	(4.90)	0.0000
30	3.17%	1,314	(4.50)	0.0000
35	3.95%	1,314	(5.26)	0.0000
40	4.32%	1,316	(5.41)	0.0000
45	4.11%	1,313	(4.92)	0.0000
50	4.25%	1,314	(4.90)	0.0000
55	4.08%	1,305	(4.27)	0.0000
60	4.17%	1,308	(4.31)	0.0000
65	5.09%	1,306	(5.01)	0.0000
70	5.03%	1,310	(4.93)	0.0000
75	5.49%	1,310	(5.28)	0.0000
80	5.44%	1,312	(5.16)	0.0000
85	5.52%	1,314	(5.25)	0.0000
90	5.55%	1,315	(5.18)	0.0000
95	5.20%	1,318	(4.88)	0.0000
99	5.52%	1,316	(5.04)	0.0000

AR Data

1% AR

days	$\mu_X - \mu_B$	df	t-val	p-val
1	(0.07%)	18,645	4.52	0.0000
2	(0.10%)	18,696	4.28	0.0000
3	(0.08%)	18,693	2.95	0.0032
4	(0.11%)	18,625	3.31	0.0009
5	(0.11%)	18,712	3.04	0.0023
10	(0.19%)	18,515	3.86	0.0001
15	(0.19%)	18,422	3.01	0.0026
20	(0.25%)	18,463	3.52	0.0004
25	(0.37%)	18,580	4.75	0.0000
30	(0.43%)	18,303	5.02	0.0000
35	(0.54%)	18,237	5.90	0.0000
40	(0.57%)	18,190	5.89	0.0000
45	(0.59%)	18,083	5.74	0.0000
50	(0.64%)	18,016	6.03	0.0000
55	(0.66%)	17,908	6.01	0.0000
60	(0.75%)	17,759	6.59	0.0000
65	(0.77%)	17,678	6.52	0.0000
70	(0.85%)	17,716	7.05	0.0000
75	(0.89%)	17,598	7.16	0.0000
80	(0.99%)	17,477	7.78	0.0000
85	(1.15%)	17,483	8.98	0.0000
90	(1.21%)	17,375	9.28	0.0000
95	(1.27%)	17,286	9.56	0.0000
99	(1.26%)	17,139	9.42	0.0000

2.5% AR

days	$\mu_X - \mu_B$	df	t-val	p-val
1	(0.10%)	9,642	5.26	0.0000
2	(0.13%)	9,604	4.72	0.0000
3	(0.12%)	9,679	3.64	0.0003
4	(0.10%)	9,621	2.72	0.0066
5	(0.12%)	9,488	2.84	0.0045
10	(0.28%)	9,467	4.58	0.0000
15	(0.34%)	9,522	4.70	0.0000
20	(0.44%)	9,587	5.27	0.0000
25	(0.58%)	9,520	6.17	0.0000
30	(0.63%)	9,478	6.18	0.0000
35	(0.72%)	9,358	6.62	0.0000
40	(0.69%)	9,398	5.99	0.0000
45	(0.80%)	9,349	6.59	0.0000
50	(0.89%)	9,207	7.03	0.0000
55	(0.95%)	9,134	7.15	0.0000
60	(1.02%)	9,051	7.44	0.0000
65	(1.08%)	9,063	7.63	0.0000
70	(1.09%)	9,038	7.51	0.0000
75	(1.14%)	8,982	7.66	0.0000
80	(1.27%)	8,967	8.39	0.0000
85	(1.41%)	8,935	9.12	0.0000
90	(1.45%)	8,893	9.23	0.0000
95	(1.49%)	8,885	9.41	0.0000
99	(1.53%)	8,811	9.50	0.0000

5% AR

days	$\mu_X - \mu_B$	df	t-val	p-val
1	(0.58%)	17	2.12	0.0487
2	(0.44%)	17	1.10	0.2859
3	(0.18%)	17	0.36	0.7248
4	(0.12%)	17	0.23	0.8200
5	(0.11%)	17	0.18	0.8558
10	(0.26%)	17	0.32	0.7542
15	0.15%	17	(0.16)	0.8751
20	0.15%	17	(0.13)	0.8980
25	(0.50%)	17	0.42	0.6795
30	(0.35%)	17	0.33	0.7443
35	(0.08%)	17	0.08	0.9344
40	(0.39%)	17	0.44	0.6690
45	(0.67%)	17	0.68	0.5077
50	(0.01%)	17	0.01	0.9917
55	0.08%	17	(0.07)	0.9436
60	(0.41%)	17	0.45	0.6599
65	(0.05%)	17	0.06	0.9505
70	(0.84%)	17	1.07	0.2983
75	(0.99%)	17	1.28	0.2163
80	(1.23%)	17	1.30	0.2116
85	(1.38%)	17	1.14	0.2697
90	(1.01%)	17	0.86	0.4006
95	(1.21%)	17	1.18	0.2533
99	(1.12%)	17	0.99	0.3360

GARCH-M Data

1% GARCH-M

days	$\mu_X - \mu_B$	df	t-val	p-val
1	(0.17%)	17,736	11.18	0.0000
2	(0.19%)	17,454	8.31	0.0000
3	(0.21%)	17,624	7.60	0.0000
4	(0.23%)	17,692	7.19	0.0000
5	(0.24%)	17,715	6.67	0.0000
10	(0.33%)	17,503	6.45	0.0000
15	(0.44%)	17,668	7.10	0.0000
20	(0.52%)	17,740	7.26	0.0000
25	(0.52%)	17,588	6.52	0.0000
30	(0.57%)	17,378	6.62	0.0000
35	(0.63%)	17,332	6.72	0.0000
40	(0.72%)	17,427	7.31	0.0000
45	(0.83%)	17,383	7.96	0.0000
50	(0.86%)	17,209	7.91	0.0000
55	(0.92%)	17,113	8.07	0.0000
60	(1.03%)	17,027	8.69	0.0000
65	(1.17%)	17,087	9.59	0.0000
70	(1.24%)	17,017	9.87	0.0000
75	(1.35%)	16,904	10.39	0.0000
80	(1.40%)	16,778	10.47	0.0000
85	(1.43%)	16,731	10.44	0.0000
90	(1.53%)	16,616	10.94	0.0000
95	(1.59%)	16,492	11.13	0.0000
99	(1.63%)	16,320	11.17	0.0000

2.5% GARCH-M

days	$\mu_X - \mu_B$	df	t-val	p-val
1	(0.29%)	3,504	9.28	0.0000
2	(0.32%)	3,494	6.95	0.0000
3	(0.34%)	3,527	6.05	0.0000
4	(0.33%)	3,513	5.10	0.0000
5	(0.32%)	3,534	4.44	0.0000
10	(0.32%)	3,530	3.15	0.0017
15	(0.30%)	3,559	2.46	0.0138
20	(0.23%)	3,591	1.70	0.0897
25	(0.09%)	3,573	0.61	0.5437
30	(0.26%)	3,599	1.62	0.1058
35	(0.24%)	3,632	1.37	0.1699
40	(0.46%)	3,685	2.59	0.0095
45	(0.51%)	3,692	2.74	0.0062
50	(0.53%)	3,696	2.73	0.0063
55	(0.62%)	3,723	3.13	0.0018
60	(0.75%)	3,753	3.74	0.0002
65	(0.90%)	3,769	4.36	0.0000
70	(1.03%)	3,789	4.90	0.0000
75	(1.15%)	3,812	5.42	0.0000
80	(1.28%)	3,860	6.01	0.0000
85	(1.36%)	3,883	6.34	0.0000
90	(1.37%)	3,882	6.27	0.0000
95	(1.46%)	3,908	6.66	0.0000
99	(1.61%)	3,921	7.28	0.0000

5% GARCH-M

days	$\mu_X - \mu_B$	df	t-val	p-val
1	(0.33%)	346	2.12	0.0350
2	(0.10%)	346	0.41	0.6799
3	(0.03%)	346	0.09	0.9294
4	(0.05%)	346	0.15	0.8826
5	0.12%	346	(0.33)	0.7382
10	0.83%	346	(1.53)	0.1261
15	2.41%	346	(3.76)	0.0002
20	3.52%	346	(4.91)	0.0000
25	3.92%	346	(4.71)	0.0000
30	4.69%	346	(5.15)	0.0000
35	5.15%	346	(5.48)	0.0000
40	6.19%	346	(6.08)	0.0000
45	7.09%	346	(6.33)	0.0000
50	7.05%	346	(6.08)	0.0000
55	7.31%	346	(6.03)	0.0000
60	7.67%	346	(5.96)	0.0000
65	8.16%	346	(6.05)	0.0000
70	7.98%	346	(6.02)	0.0000
75	8.76%	346	(6.49)	0.0000
80	8.88%	346	(6.38)	0.0000
85	8.82%	346	(6.18)	0.0000
90	9.22%	346	(6.28)	0.0000
95	9.70%	346	(6.55)	0.0000
99	9.40%	346	(6.31)	0.0000

10% GARCH-M

days	$\mu_X - \mu_B$	df	t-val	p-val
1	(1.42%)	17	1.01	0.3258
2	(1.24%)	17	0.73	0.4745
3	(0.20%)	17	0.07	0.9440
4	1.03%	17	(0.34)	0.7408
5	1.18%	17	(0.42)	0.6818
10	5.79%	17	(1.78)	0.0929
15	12.99%	17	(2.77)	0.0132
20	15.60%	17	(2.82)	0.0118
25	12.04%	17	(1.95)	0.0677
30	8.99%	17	(1.50)	0.1511
35	12.34%	17	(1.93)	0.0699
40	11.49%	17	(1.79)	0.0914
45	15.00%	17	(2.06)	0.0551
50	16.67%	17	(2.32)	0.0327
55	19.49%	17	(2.70)	0.0153
60	21.62%	17	(2.86)	0.0108
65	19.69%	17	(2.89)	0.0102
70	22.68%	17	(3.18)	0.0055
75	21.28%	17	(3.15)	0.0059
80	19.80%	17	(2.96)	0.0087
85	18.08%	17	(2.67)	0.0160
90	16.02%	17	(2.25)	0.0380
95	15.84%	17	(2.18)	0.0437
99	16.90%	17	(2.21)	0.0407

EGARCH Data

1% EGARCH

days	$\mu_X - \mu_B$	df	t-val	p-val
1	(0.18%)	17,717	11.34	0.0000
2	(0.18%)	17,436	7.31	0.0000
3	(0.17%)	17,791	5.83	0.0000
4	(0.16%)	17,672	4.51	0.0000
5	(0.15%)	17,687	3.83	0.0001
10	(0.20%)	17,585	3.61	0.0003
15	(0.24%)	17,633	3.71	0.0002
20	(0.36%)	17,480	4.80	0.0000
25	(0.46%)	17,472	5.58	0.0000
30	(0.50%)	17,461	5.67	0.0000
35	(0.59%)	17,356	6.36	0.0000
40	(0.66%)	17,288	6.69	0.0000
45	(0.77%)	17,024	7.43	0.0000
50	(0.81%)	16,962	7.48	0.0000
55	(0.99%)	16,898	8.78	0.0000
60	(1.05%)	16,663	9.02	0.0000
65	(1.18%)	16,522	9.85	0.0000
70	(1.32%)	16,362	10.79	0.0000
75	(1.35%)	16,327	10.79	0.0000
80	(1.49%)	16,283	11.63	0.0000
85	(1.59%)	16,081	12.15	0.0000
90	(1.75%)	15,941	13.09	0.0000
95	(1.82%)	15,855	13.46	0.0000
99	(1.85%)	15,786	13.52	0.0000

2.5% EGARCH

days	$\mu_X - \mu_B$	df	t-val	p-val
1	(0.30%)	5,446	11.16	0.0000
2	(0.27%)	5,467	6.90	0.0000
3	(0.26%)	5,426	5.29	0.0000
4	(0.22%)	5,436	3.81	0.0001
5	(0.16%)	5,475	2.45	0.0142
10	0.14%	5,515	(1.56)	0.1195
15	0.22%	5,518	(2.10)	0.0354
20	0.20%	5,536	(1.70)	0.0893
25	0.23%	5,610	(1.81)	0.0699
30	0.37%	5,630	(2.67)	0.0076
35	0.39%	5,613	(2.65)	0.0081
40	0.38%	5,631	(2.49)	0.0129
45	0.34%	5,617	(2.14)	0.0322
50	0.30%	5,613	(1.77)	0.0763
55	0.21%	5,623	(1.23)	0.2178
60	0.08%	5,588	(0.43)	0.6672
65	(0.01%)	5,552	0.05	0.9562
70	(0.02%)	5,547	0.09	0.9258
75	(0.07%)	5,538	0.36	0.7171
80	(0.10%)	5,556	0.52	0.6062
85	(0.09%)	5,533	0.44	0.6593
90	(0.17%)	5,483	0.83	0.4075
95	(0.25%)	5,471	1.24	0.2138
99	(0.38%)	5,463	1.85	0.0641

5% EGARCH

days	$\mu_X - \mu_B$	df	t-val	p-val
1	(0.53%)	356	3.82	0.0002
2	(0.26%)	356	1.28	0.2004
3	(0.05%)	357	0.21	0.8330
4	0.24%	357	(0.81)	0.4157
5	0.21%	357	(0.64)	0.5219
10	1.05%	357	(2.30)	0.0223
15	1.65%	357	(3.00)	0.0029
20	2.33%	357	(3.75)	0.0002
25	2.59%	357	(3.88)	0.0001
30	3.07%	357	(4.45)	0.0000
35	3.07%	357	(4.31)	0.0000
40	2.97%	358	(4.10)	0.0001
45	2.76%	358	(3.72)	0.0002
50	2.95%	358	(3.93)	0.0001
55	3.24%	358	(4.19)	0.0000
60	3.29%	358	(4.24)	0.0000
65	3.15%	358	(3.98)	0.0001
70	3.15%	358	(3.84)	0.0001
75	3.39%	358	(4.04)	0.0001
80	3.15%	358	(3.66)	0.0003
85	2.71%	358	(3.14)	0.0018
90	2.72%	358	(3.15)	0.0018
95	2.96%	358	(3.36)	0.0009
99	3.01%	358	(3.43)	0.0007

Comparing the real market data with that of the various models using the [redacted] down technical analysis, we find vast differences between the two, with the exception of the large moves in the EGARCH model. The AR models predict excess negative returns, where as the real data point to excess positive returns following large down moves. The GARCH-M model, once again, predicts overly large excess positive returns, however these do not have the same high level of statistical significance in the 10% moves region due to the small amount of data and could be a statistical anomaly.

However, it is interesting to point out the EGARCH model has similar return patterns for 5% [redacted] as the market data, albeit again a little larger in magnitude. This is surprising because under EMH (even the weak theory), one is not supposed to be able to predict market returns using only technical tools, yet as we clearly see, these set of technical tools fit within EMH, at least in one of the models.

This initial, exploratory research offers interesting avenues for further work. These patterns should be examined in other samples of equities, as well as in other instruments (debt and derivatives) and timeframes, and we need to better understand the relationship between the EMH models and these returns. Though what we have done here does not involve charts, "indicators" and complex price patters, we believe this work represents one aspect of Technical Analysis. If these patterns appear in other securities, they may offer opportunities for above average trading profits and could offer strong support for the ability to predict some aspects of future price paths based on past prices.